The Net Change Theorem:

It's an application of 2nd FTC, $\displaystyle\int_a^b F'(x)\,dx = \underbrace{F(b)-F(a)}_{\text{net change}}$

Recall: Velocity, $v(t) = s'(t)$, where $s(t)$ is displacement

Then we have, $\displaystyle\int_a^b v(t)\,dt = \underbrace{s(b)-s(a)}_{\substack{\text{net displacement} \\ \text{between time } a \text{ to } b.}}$

Example:- The velocity of a decelerating car is given by $v(t) = -\frac{1}{5}t + 20$. How far does the car travel between $t=0$ & $t=100$?

Solution:

$$\int_0^{100} v(t)\,dt = \int_0^{100}\left(-\frac{1}{5}t + 20\right)dt$$

$$= \left[-\frac{t^2}{10} + 20t\right]_0^{100}$$

$$= \left(-\frac{100^2}{10} + 20(100)\right) - \left(-\frac{0^2}{10} + 20(0)\right)$$

$$= 1000$$

More examples related to net change:

① Compute: $\displaystyle\int_0^8 e^{-x}\,dx$

Note:- $\frac{d}{dx}\left(-e^{-x}\right) = -\left(e^{-x}\cdot(-1)\right) = e^{-x}$

So, antiderivative of e^{-x} is $-e^{-x}$.

Hence, $\int_0^8 e^{-x}\,dx = \left[-e^{-x}\right]_0^8 = (-e^{-8}) - (-e^{-0})$

$= 1 - e^{-8}$

Note :- Total distance : $\int_a^b |v(t)|\,dt$.

② Suppose $v(t) = (t-1)^2(t-2)$ over $0 \le t \le 3$.

(a) What's the net displacement of the object between the interval $[0,3]$?

(b) What's the total displacement of the object between the interval $[0,3]$?

→ (a) Net displacement $= \int_0^3 v(t)\,dt$

$(t-1)^2(t-2)$
$= (t^2-2t+1)(t-2)$
$= t^3-4t^2+5t-2$

$= \int_0^3 (t-1)^2(t-2)\,dt$

$= \int_0^3 (t^3-4t^2+5t-2)\,dt$

$= \left[\frac{t^4}{4} - \frac{4t^3}{3} + \frac{5t^2}{2} - 2t\right]_0^3$

$= \left(\frac{3^4}{4} - \frac{4\cdot 3^3}{3} + \frac{5\cdot 3^2}{2} - 2\cdot 3\right) - (0)$

$= \frac{81}{4} - 42 + \frac{45}{2}$

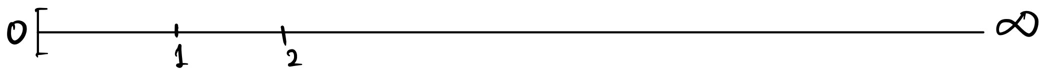
$= \cancel{20} + \frac{1}{4} - \cancel{42} + \cancel{22} + \frac{1}{2}$

$= \frac{3}{4}$

(b) We need $|v(t)|$, for that we need in which intervals $v(t)$ is +ve & where it is −ve.

Note:- $v(t) = 0 \Rightarrow (t-1)^2(t-2) = 0$
$\Rightarrow t = 1, 2$

Since, t represent time, it's $t \geq 0$.



For $0 \leq t < 1$, $v(t) < 0$
$1 < t < 2$, $v(t) < 0$] the object is moving backward.
$2 < t < \infty$, $v(t) > 0$] the object is moving forward.

Total Distance = backward distance between $t=0$ & $t=2$
+ forward distance between $t=2$ & $t=3$.

$$= \int_0^2 -v(t)\,dt + \int_2^3 v(t)\,dt$$

$$= \int_0^2 -(t-1)^2(t-2)\,dt + \int_2^3 (t-1)^2(t-2)\,dt$$

$$= \int_0^2 -(t^3-4t^2+5t-2)\,dt + \int_2^3 (t^3-4t^2+5t-2)\,dt$$

$$= -\left[\frac{t^4}{4} - \frac{4t^3}{3} + \frac{5t^2}{2} - 2t\right]_0^2 + \left[\frac{t^4}{4} - \frac{4t^3}{3} + \frac{5t^2}{2} - 2t\right]_2^3$$

$$= \frac{2}{3} + \frac{17}{12}$$

$$= \frac{25}{12}$$